# Problem Solving – march 2020_2 – Solutions

## Some Useful Information

[1] A problem from last week was: What is the sum of the integers from 1 to 100, inclusive?

In an *arithmetic series* the difference between terms is a constant. The formula for the sum is:

*n (a+z) / 2*

where *n* is the number of terms in the sequence, *a* is the lowest term, and *z* is the highest term. Therefore, the sum of our sequence = 100 (1 + 100) / 2 → 5050

[2] A **triangular number** is a figurate number that can be represented in the form of a triangular grid of points where the first row contains a single element and each subsequent row contains one more element than the previous one. 1, 3, 6, 10, 15, 21, 28, 36, 45, 55 are the first eleven **triangular** numbers. To find the nth triangular number, one can use the following formula:

*n (n + 1) / 2*

Example: Find the $28^{th}$ triangular number. 28 (28 + 1) / 2 = 406

[3] There are 25 prime numbers less than 100:
2,3,5,7,11,13,17,19,23,29,31,37,41,43,47,53,59,61,67,71,73,79,83,89,97

There are 21 prime numbers between 101 and 200:
101,103,107,109,113,127,131,137,139,149,151,157,163,167,173,179,181,191,193,197,199

There are 16 prime numbers between 201 and 300:
211,223,227,229,233,239,241,251,257,263,269,271,277,281,283,293

## Practice Problems

[1] What is the sum of the even integers from 1 to 100, inclusive? ***2550***

[2] If the sum of the positive integer a and 5 is less than 12, what is the sum of all possible values of a? ***21***

[3] Robin rolls two fair octahedral dice, each with faces numbered 1 through 8. What is the probability that the sum of the numbers she rolls is 8? Express your answer as a common fraction. ***7/64***

[4] What is the greatest two-digit prime number whose digits are both prime numbers? ***73***

[5] A bundle of 25 one-dollar bills weighs 9/10 of an ounce. How many pounds would 4000 one-dollar bills weigh? ***9 lbs***

[6] How many pairs of positive integers *x* and *y* are solutions of: *x*/12 + *y*/36 = 1? ***11 pairs***

[7] The arithmetic mean of ten numbers is 37. What number can be added to the set so that the arithmetic mean of the eleven numbers is 41? ***81***

[8] What is the least positive integer that contains each of the digits from 1 to 3 at least once and is divisible by 9? ***1233***

[9] If *a*, *b* and *c* are positive integers such that $a + b + c = 7$, what is the least possible value of $a! + b! + c!$? (e.g. 5! = 5x4x3x2x1) ***10 (2,2,3)***

[10] What is the sum of all four-digit positive integers that contain each of the digits 6, 7, 8 and 9? ***199,980 (6x30x1111)***

[11] A person who has *a* quarters and *b* nickels has $5.60 more than a person who has *a* nickels and *b* quarters. What is the value of $a - b$? ***28***

[12] What positive value should replace *y* in this statement to make it true? ***15***

$$55 \times 59 - 53 \times 57 = y^2 - 1$$

[13] The Pythagoreans proved this pattern: $1^2 = 1$; $2^2 = 1 + 3$; $3^2 = 1 + 3 + 5$; $4^2 = 1 + 3 + 5 + 7$

Nicomachus discovered this pattern: $1^3 = 1$; $2^3 = 3 + 5$; $3^3 = 7 + 9 + 11$; $4^3 = 13 + 15 + 17 + 19$

What is the value of *s* in this equation: $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 = s^2$ ***36***

[14] Find the smallest whole number that is a common multiple of 18 and 40 and is also a perfect square. ***3600***

[15] Create a 10-digit number so that the first digit tells you how many 0's are in the number, the second digit tells you how many 1's are in the number, the third digit tells you how many 2's are in the number, etc. ***6210001000***

___ ___ ___ ___ ___ ___ ___ ___ ___ ___

0's 1's 2's 3's 4's 5's 6's 7's 8's 9's

## Practice Meet

[1] Sheila's uncle gave her a bag of jellybeans for her birthday. On the first day, she ate 15 jellybeans. On the second day, she shared 2/3 of the remaining jellybeans with her friends who came to her birthday party. After the party, she saved 7 jellybeans for herself and then split the remaining jellybeans between her two brothers. Each brother got 10 jellybeans. How many ***dozen*** jellybeans did Sheila receive from her uncle? ***8 dozen***

[2] There are 100 students in the 9th grade and each student is assigned a locker. The 100 students start down the hall one at a time. The first student opens every locker. The second student closes all the lockers that are multiples of 2. The third student changes the lockers that are multiples of 3 (which means if a locker is closed, they open it. If the locker is open, they close it.) The fourth student changes all the lockers that are multiples of 4. This continues until the 100th student changes the 100th locker. Then, the principal walks down the hall, and changes all the lockers that are prime numbers.

(a) At the end, how many lockers will be closed? ***65 (100 – 10 – 25)***

(b) In another school there are 300 ninth graders. If they did the same activity as above, how many lockers would be open at the end? ***79*** (17+62)